Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 A two-dimensional cross section is taken of a three-dimensional object. If this cross section is a triangle, what can *not* be the three-dimensional object?

(1) cone
(2) cylinder
(3) pyramid
(4) rectangular prism

2 The image of $\triangle DEF$ is $\triangle D'E'F'$. Under which transformation will the triangles *not* be congruent?

(1) a reflection through the origin
(2) a reflection over the line $y = x$
(3) a dilation with a scale factor of 1 centered at (2,3)
(4) a dilation with a scale factor of $\frac{3}{2}$ centered at the origin

3 The vertices of square $RSTV$ have coordinates $R(-1,5)$, $S(-3,1)$, $T(-7,3)$, and $V(-5,7)$. What is the perimeter of $RSTV$?

(1) $\sqrt{20}$
(2) $\sqrt{40}$
(3) $4\sqrt{20}$
(4) $4\sqrt{40}$

$RS = \sqrt{(-1--3)^2+(5-1)^2}$

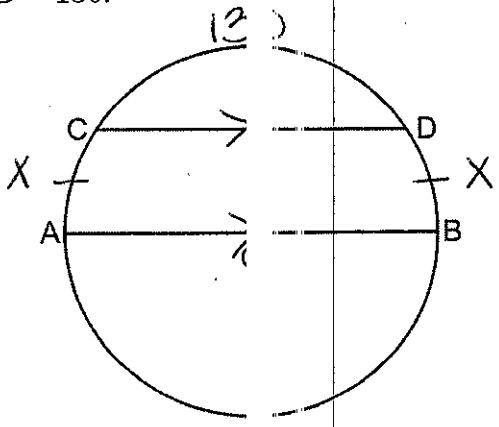
$\sqrt{(2)^2+(4)^2}$

$\sqrt{4+16}$

$\sqrt{20}$

Use this space for computations.

4 In the diagram below of circle O, chord $\overline{CD}$ is parallel to diameter $\overline{ACB}$ and $m\overset{\frown}{CD} = 130$.



$2x + 130 = 180$

$2x = 50$

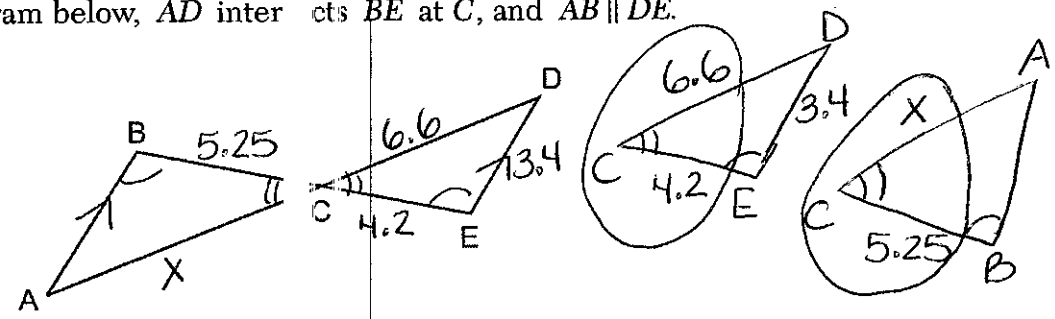
$x = 25$

What is $m\overset{\frown}{AC}$?

(1) 25 (3) 65

(2) 50 (4) 115

5 In the diagram below, $\overline{AD}$ intersects $\overline{BE}$ at C, and $\overline{AB} \parallel \overline{DE}$.



If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of $\overline{AC}$, to the *nearest hundredth of a centimeter*?

(1) 2.70 (3) 5.28

(2) 3.34 (4) 8.25

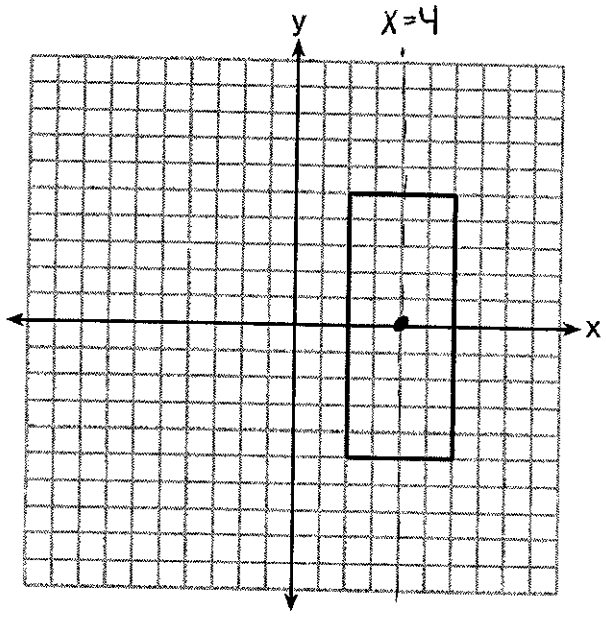
$\frac{6.6}{4.2} = \frac{x}{5.25}$

$\frac{4.2x}{4.2} = \frac{34.65}{4.2}$

$x = 8.25$

Use this space for computations.

6 As shown in the graph below, the quadrilateral is a rectangle.



Which transformation would *not* map the rectangle onto itself?

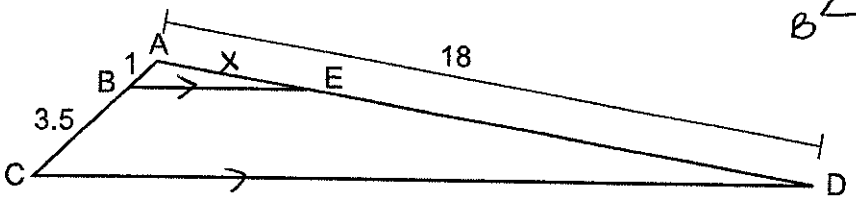
(1) a reflection over the x-axis

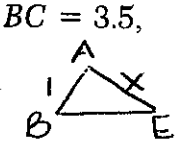
(2) a reflection over the line $x = 4$

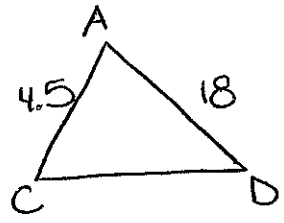
(3) a rotation of 180° about the origin

(4) a rotation of 180° about the point (4,0)

7 In the diagram below, triangle ACD has points B and E on sides $\overline{AC}$ and $\overline{AD}$, respectively, such that $\overline{BE} \parallel \overline{CD}$, $AB = 1$, $BC = 3.5$, and $AD = 18$.







What is the length of $\overline{AE}$, to the *nearest tenth*?

(1) 14.0

(2) 5.1

(3) 3.3

(4) 4.0

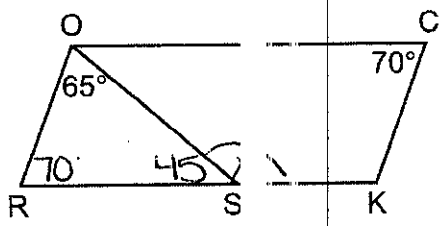
$\frac{1}{4.5} = \frac{X}{18}$

$4.5X = 18$

$X = 4.0$

Use this space for computations.

8 In the diagram below of parallelogram *ROCK*, $m\angle C$ is 70° and $m\angle ROS$ is 65°.

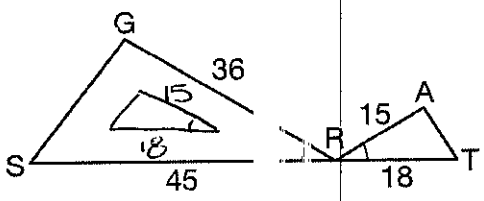


180-70-65=45
180-45=135

What is $m\angle KSO$?

(1) 45°
(2) 110°
(3) 115°
(4) 135°

9 In the diagram below, $\angle GRS \cong \angle ART$, $GR = 36$, $SR = 45$, $AR = 15$, and $RT = 18$.



$\frac{15}{36} = \frac{18}{45}$
$675 \neq 648$

Which triangle similarity statement is correct?

(1) $\triangle GRS \sim \triangle ART$ by AA.
(2) $\triangle GRS \sim \triangle ART$ by SAS.
(3) $\triangle GRS \sim \triangle ART$ by SSS.
(4) $\triangle GRS$ is not similar to $\triangle ART$.

10 The line represented by the equation $4y = 3x + 7$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

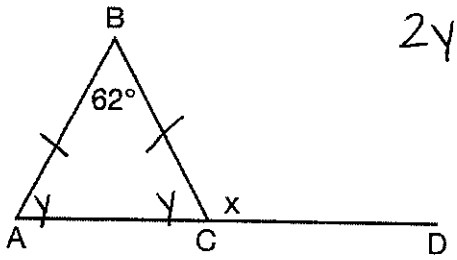
(1) $3x - 4y = 9$
(2) $3x + 4y = 9$
(3) $4x - 3y = 9$
(4) $4x + 3y = 9$

$4y = 3x + 7$
$-3x \quad -3x$
$-3x + 4y = 7$

not on the line - dilated line is parallel

Use this space for computations.

11 Given $\triangle ABC$ with $m\angle B = 62°$ and side $\overline{AC}$ extended to D, as shown below.



$2y + 62 = 180$

$\frac{2y}{2} = \frac{118}{2}$

$y = 59$

$180 - 59 = 121$

Which value of x makes $\overline{AB} \cong \overline{CB}$?

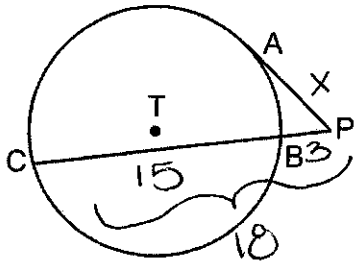
(1) 59°

(2) 62°

(3) 118°

● 121°

12 In the diagram shown below, $\overline{PA}$ is tangent to circle T at A, and secant $\overline{PBC}$ is drawn where point B is on circle T.



$x^2 = 3(18)$

$\sqrt{x^2} = \sqrt{54}$

$\sqrt{9}\sqrt{6}$

$3\sqrt{6}$

If $PB = 3$ and $BC = 15$, what is the length of $\overline{PA}$?

(1) $3\sqrt{5}$

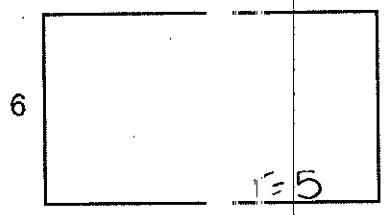
● $3\sqrt{6}$

(3) 3

(4) 9

Use this space for computations.

13 A rectangle whose length and w th are 10 and 6, respectively, is shown below. The rectangle is cor nuously rotated around a straight line to form an object whose volu e is 150π.



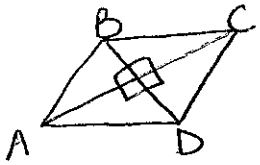
$\pi r^2 h$
$\pi(5)^2(6)$
150π

Which line could the rectangle b rotated around?

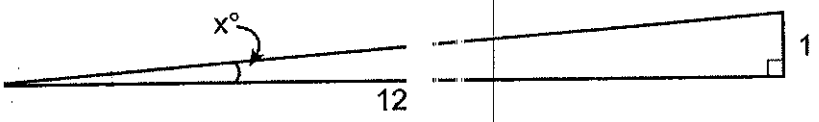
(1) a long side
(2) a short side
(3) the vertical line of symmetry
(4) the horizontal line of symmetry

14 If $ABCD$ is a parallelogram, w ich statement would prove that $ABCD$ is a rhombus?

(1) $\angle ABC \cong \angle CDA$
(2) $\overline{AC} \cong \overline{BD}$
(3) $\overline{AC} \perp \overline{BD}$
(4) $\overline{AB} \perp \overline{CD}$



15 To build a handicapped-acces ramp, the building code states that for every 1 inch of vertical r e in height, the ramp must extend out 12 inches horizontally, as sh vn in the diagram below.



$\tan X = \frac{1}{12}$

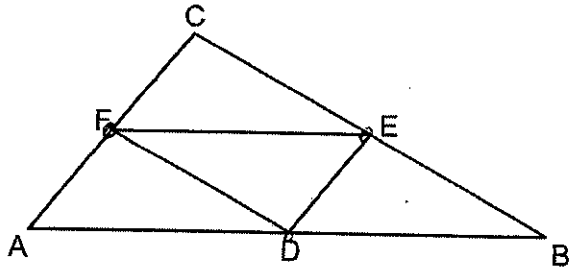
$X = 4.76$

What is the angle of inclinatic , x, of this ramp, to the *nearest hundredth of a degree*?

(1) 4.76
(2) 4.78
(3) 85.22
(4) 85.24

Use this space for computations.

16 In the diagram below of $\triangle ABC$, D, E, and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively.



What is the ratio of the area of $\triangle CFE$ to the area of $\triangle CAB$?

(1) 1:1
(2) 1:2
(3) 1:3
(4) 1:4

17 The coordinates of the endpoints of $\overline{AB}$ are $A(-8,-2)$ and $B(16,6)$. Point P is on $\overline{AB}$. What are the coordinates of point P, such that $AP:PB$ is 3:5?

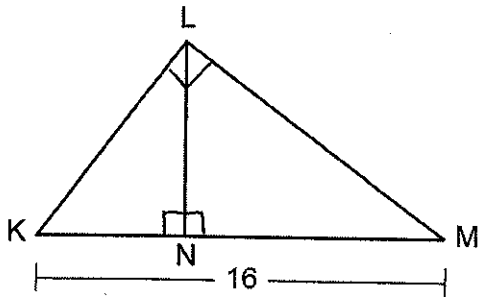
(1) (1,1)
(2) (7,3)
(3) (9.6,3.6)
(4) (6.4,2.8)

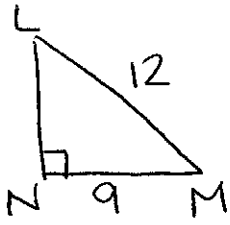
$A(-8,-2) \xrightarrow{T\langle 24,8\rangle} B(16,6)$

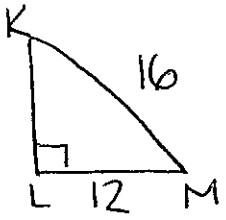
$24\left(\frac{3}{8}\right)\ 8\left(\frac{3}{8}\right)$

$A(-8,-2) \xrightarrow{T\langle 9,3\rangle} P(1,1)$

18 Kirstie is testing values that would make triangle KLM a right triangle when $\overline{LN}$ is an altitude, and $KM = 16$, as shown below.







$\frac{12}{16} = \frac{9}{12}$

$144 = 144$

Which lengths would make triangle KLM a right triangle?

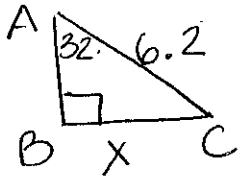
(1) $LM = 13$ and $KN = 6$
(2) $LM = 12$ and $NM = 9$
(3) $KL = 11$ and $KN = 7$
(4) $LN = 8$ and $NM = 10$

Use this space for computations.

19 In right triangle ABC, $m\angle A = 32$ $m\angle B = 90°$, and $AC = 6.2$ cm. What is the length of $\overline{BC}$, to the *arest tenth of a centimeter*?

(1) 3.3 (3) 5.3

(2) 3.9 (4) 11.7



Sin 32 = $\frac{X}{6.2}$

20 The 2010 U.S. Census populati ns and population densities are shown in the table below.

PD = $\frac{\text{people}}{\text{area}}$

State	Population D sity $\left(\frac{\text{people}}{\text{mi}^2}\right)$	Population in 2010
Florida	35 .6	18,801,310
Illinois	23 .1	12,830,632
New York	4' .2	19,378,102
Pennsylvania	28 .9	12,702,379

53626.09812
55519.82691
47125.73444
44742.441

Based on the table above, which st has the states' areas, in square miles, in order from largest to sr llest?

(1) Illinois, Florida, New York, I nsylvania

(2) New York, Florida, Illinois, I nsylvania

(3) New York, Florida, Pennsylv nia, Illinois

(4) Pennsylvania, New York, Flc da, Illinois

21 In a right triangle, $\sin(40 - x)°$ cos $(3x)°$. What is the value of x?

(1) 10 (3) 20

(2) 15 (4) 25

$40 - X + 3X = 90$
$40 + 2X = 90$
$2X = 50$
$X = 25$

22 A regular decagon is rotated n legrees about its center, carrying the decagon onto itself. The val of n could be

(1) 10° (3) 225°

(2) 150° (4) 252°

$\frac{360}{10} = 36, 72, 108, 144, 180, 216, 2$

Use this space for computations.

23 In a circle with a diameter of 32, the area of a sector is $\frac{512\pi}{3}$. The measure of the angle of the sector, in radians, is

(1) $\frac{\pi}{3}$

(3) $\frac{16\pi}{3}$

● $\frac{4\pi}{3}$

(4) $\frac{64\pi}{3}$

$A.S. = \frac{n}{360} \cdot \pi r^2$

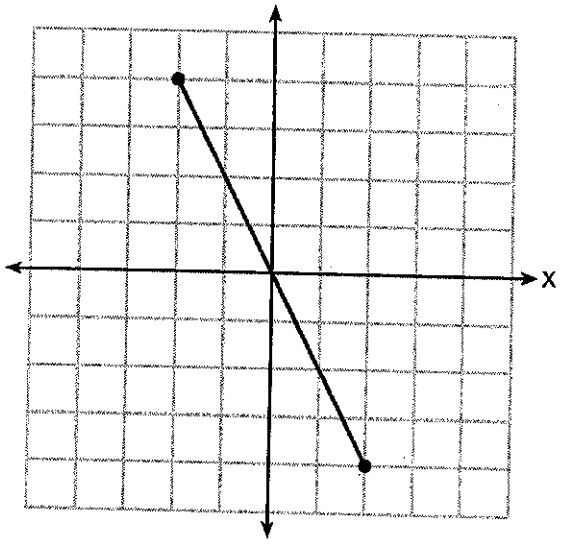
$\frac{512\pi}{3} = \frac{n(16)^2\pi}{360}$

$\frac{184320\pi}{768\pi} = \frac{768\pi n}{768\pi}$

$240 = n$

$240\left(\frac{\pi}{180}\right) = \frac{4\pi}{3}$

24 What is an equation of the perpendicular bisector of the line segment shown in the diagram below?



$m = -2$

$\perp m = \frac{1}{2}$ pt (0,0)

$y - y = m(x - x)$

$y - 0 = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x$

$2y = x$

$2y - x = 0$

(1) $y + 2x = 0$

(2) $y - 2x = 0$

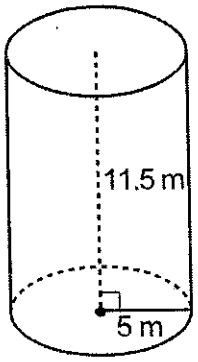
(3) $2y + x = 0$

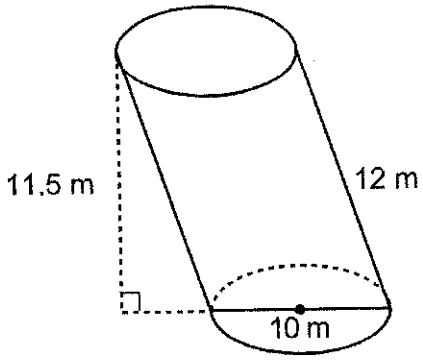
● $2y - x = 0$

Part II

Answer all 7 questions in thi part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, incl ding appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the informati n provided for each question to determine your answer. Note that diagrams are not neces ri'y drawn to scale. For all questions in this part, a correct numerical answer with no work hown will receive only 1 credit. All answers should be written in pen, except for graphs nd drawings, which should be done in pencil. [14]

25 Sue believes that the two cylinc rs shown in the diagram below have equal volumes.





Is Sue correct? Explain why.

Base Area πr^2 πr^2

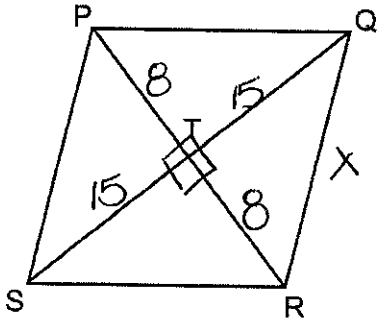
$\pi(5)^2$ $\pi(5)^2$

$25\pi = 25\pi$

Height $11.5 = 11.5$

Cavalieri's Principle states if the base areas are equal and the heights are equal then the volumes are equal

26 In the diagram of rhombus $PQRS$ below, the diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point T, $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.



Diagonals ⊥
Diagonals bisect each other

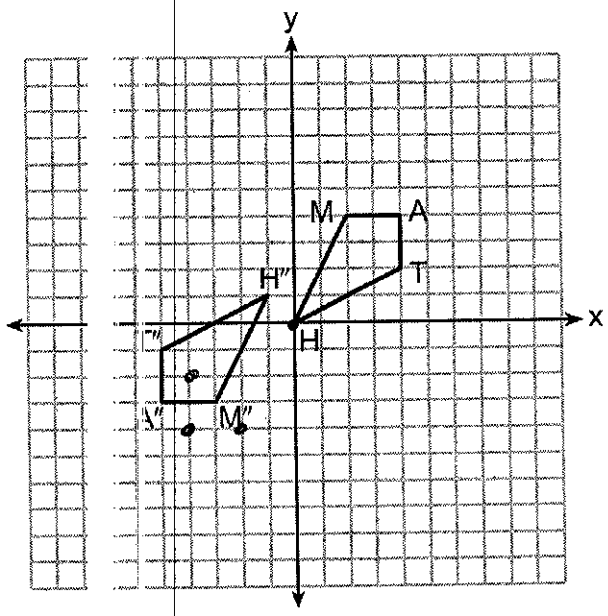
$8^2 + 15^2 = X^2$

$\sqrt{289} = \sqrt{X^2}$

$17 = X$

$4(17) = \boxed{68}$

27 Quadrilateral *MATH* and its image *M″A″T″H″* are graphed on the set of axes below.

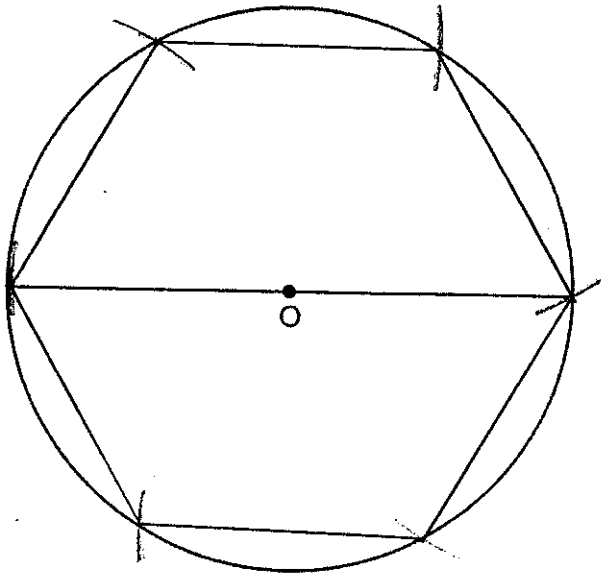


Describe a sequence of transformations that maps quadrilateral *MATH* onto quadrilateral *M″A″T″H″*.

A rotation of 180° about the origin

A translation left 1 and up 1

28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O.
[Leave all construction marks.]



29 The coordinates of the endpoi s of $\overline{AB}$ are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilatic of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes belc is optional.]

$D_{\frac{1}{2}}$

$A(2,3) \rightarrow A'(1, 1.5)$

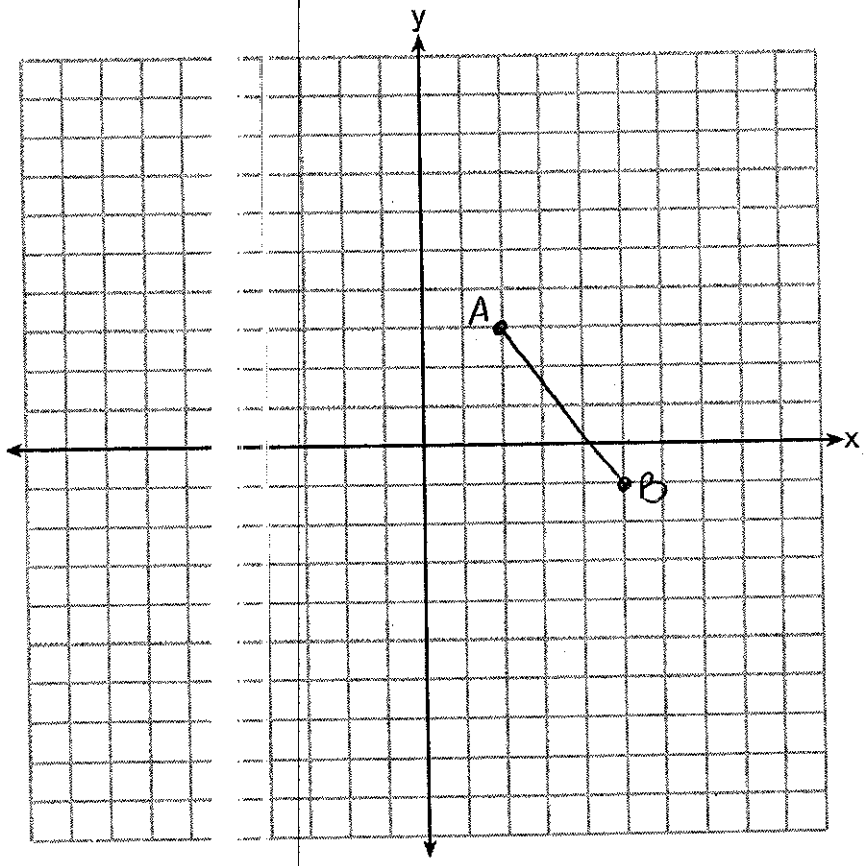
$B(5,-1) \rightarrow B'(2.5, -.5)$

$\sqrt{(1-2.5)^2+(1.5-(-.5))^2}$

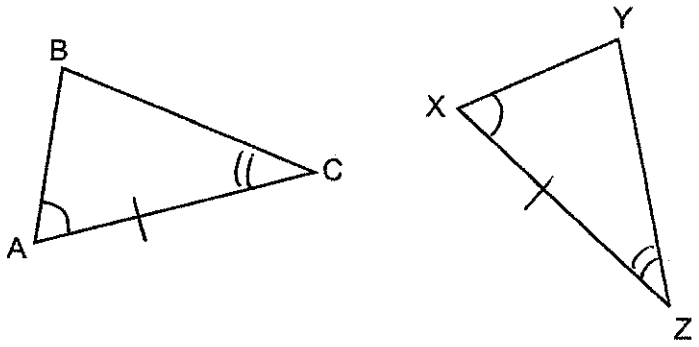
$\sqrt{(-1.5)^2+(2)^2}$

$\sqrt{2.25+4}$

$\sqrt{6.25} = \boxed{2.5}$



30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.



Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

$\overline{BC} \cong \overline{YZ}$ b/c the $\triangle$s are $\cong$ by ASA

$\angle A \cong \angle X$, $\angle C \cong \angle Z$, $\overline{AC} \cong \overline{XZ}$

Rigid Motions preserve distance (lengths) and angle measures

Since $\triangle ABC \cong \triangle XYZ$ then $\overline{BC} \cong \overline{YZ}$ b/c of CPCTC

31 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + y^2 - 6x = 56 + 8y$.

$8y$

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9 \quad \left(\frac{8}{2}\right)^2 = (4)^2 = 16$

$x^2 - 6x + 9 + y^2 + 8y + 16 = 56 + 9 + 16$

$(x-3)^2 + (y+4)^2 = 81$

$C = (3, -4)$

$\sqrt{r^2} = \sqrt{81}$

$r = 9$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of $\overline{PQ}$ and $\overline{RQ}$, respectively. Use coordinate geometry to prove that $\overline{AB}$ is parallel to $\overline{PR}$ and is half the length of $\overline{PR}$.

[The use of the set of axes below is optional.]

mdpt PQ $\left(\frac{-3+-1}{2}, \frac{-1+7}{2}\right)$

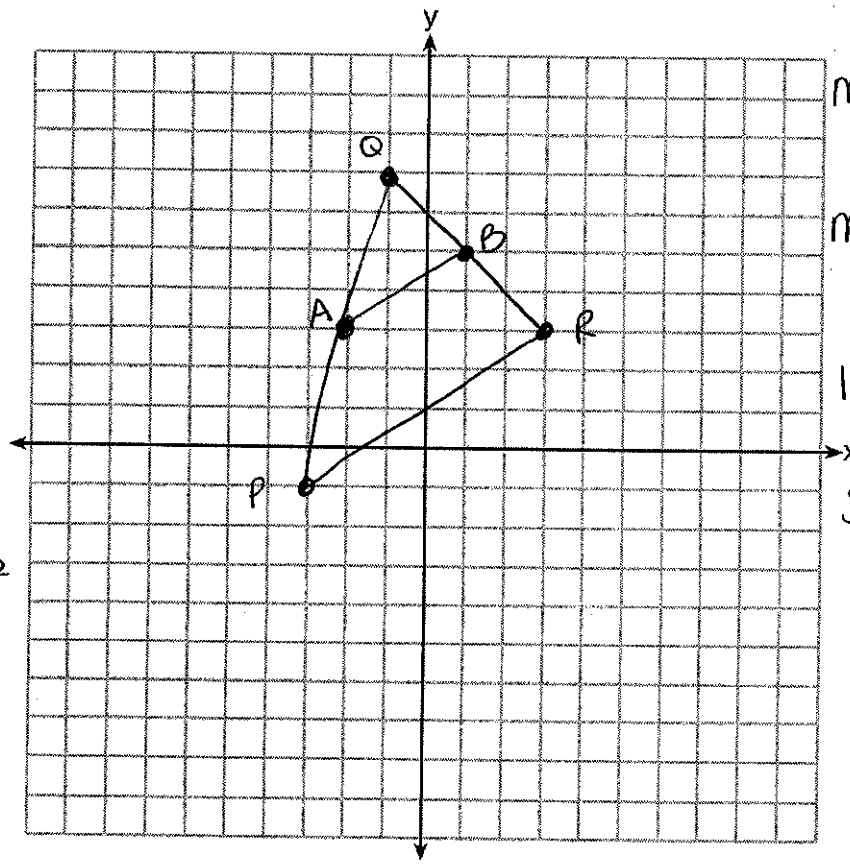
$\left(\frac{-4}{2}, \frac{6}{2}\right)$

$A=(-2,3)$

mdpt QR $\left(\frac{-1+3}{2}, \frac{7+3}{2}\right)$

$\left(\frac{2}{2}, \frac{10}{2}\right)$

$B=(1,5)$



$AB=\sqrt{(-2-1)^2+(3-5)^2}$

$\sqrt{(-3)^2+(-2)^2}$

$\sqrt{9+4}$

$\sqrt{13}$

$PR=\sqrt{(-3-3)^2+(-1-3)^2}$

$\sqrt{(-6)^2+(-4)^2}$

$\sqrt{36+16}$

$\sqrt{52}$

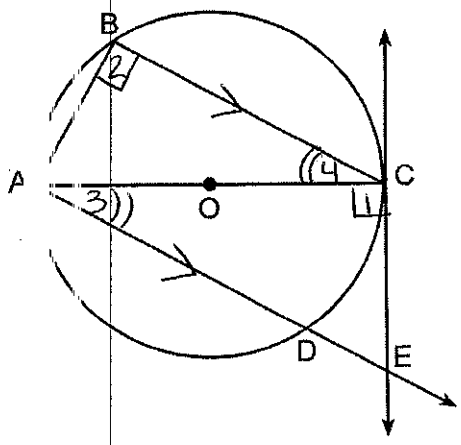
$m_{AB}=\frac{2}{3}$

$m_{PR}=\frac{4}{6}=\frac{2}{3}$

11 lines have the same slope

$\frac{\sqrt{52}}{2}=\sqrt{13}$ ✓

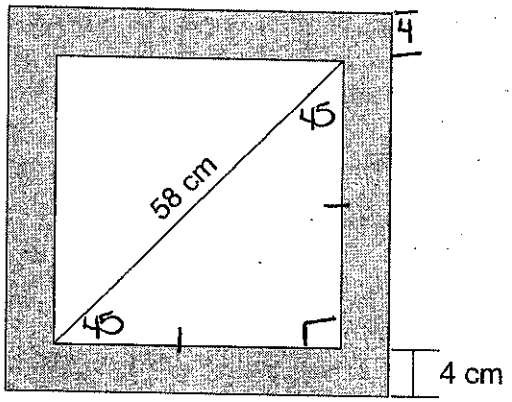
33 In the diagram below of circle , tangent $\overline{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overline{ADE}$, and chord $\overline{AB}$ is drawn.



Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

S	R
①Tangent EC drawn to diameter AC	①Given
②AC ⊥ CE	②Tangents are ⊥ to a diameter at the point of tangency
③∠1 is a right ∠	③⊥ lines form right ∠s
④∠2 is a right ∠	④An ∠ inscribed in a semi-circle forms a right ∠
⑤∠1 ≅ ∠2	⑤All right ∠s are ≅
⑥Chord BC ∥ secant ADE	⑥Given
⑦∠3 ≅ ∠4	⑦∥ lines cut by a transversal form ≅ alternate interior ∠s
⑧△ABC ~ △ECA	⑧AA ~
⑨$\frac{BC}{CA} = \frac{AB}{EC}$	⑨Corresponding sides of ~△s are in proportion

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

$\sin 45 = \frac{X}{58}$

$X = 41.01219331$

$+ \quad 8$

49.01219331

$A = lw$

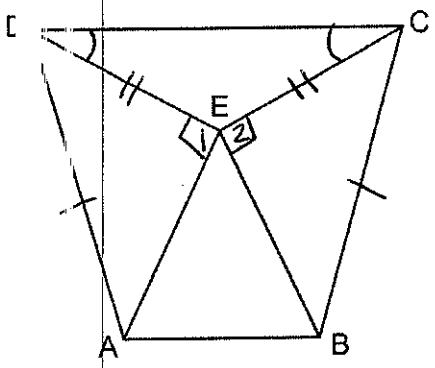
$A = (49.01219331)(49.01219331)$

$\boxed{A = 2402.2}$

Part IV

Answer the 2 questions in th part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, incl ling appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the informati n provided for each question to determine your answer. Note that diagrams are not neces: ri y drawn to scale. For all questions in this part, a correct numerical answer with no work hown will receive only 1 credit. All answers should be written in pen, except for graphs nd drawings, which should be done in pencil. [12]

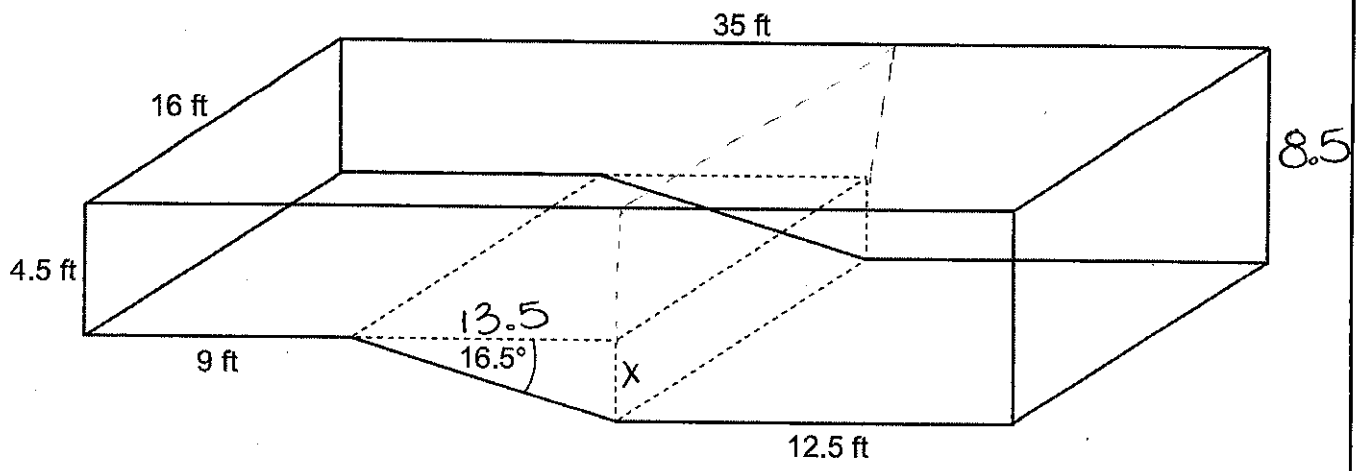
35 Isosceles trapezoid $ABCD$ has l ses $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn n rapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



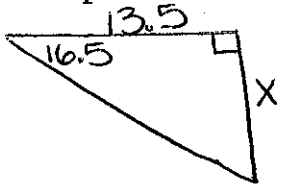
Prove $\triangle ADE \cong \triangle BCE$ and p ve $\triangle AEB$ is an isosceles triangle.

S	R
① Isosceles Trapezoid ABCD non parallel legs $\overline{AD}$ $\overline{BC}$ $\angle CDE \cong \angle DCE$ $\overline{AE} \perp DE$, $\overline{BE} \perp \overline{CE}$	① Given
② $\overline{AD} \cong \overline{CB}$	② In an isosceles trapezoid legs are $\cong$
③ $DE \cong CE$	③ In a $\triangle$ if 2 $\angle$s are $\cong$ then the sides opposite them are $\cong$
④ $\angle 1$ and $\angle 2$ are ric ht $\angle$s	④ $\perp$ lines form right $\angle$s
⑤ $\angle 1 \cong \angle 2$	⑤ All right $\angle$s are $\cong$
⑥ $\triangle ADE$ & $\triangle BCE$ are g $\triangle$s	⑥ Right $\triangle$s have 1 right $\angle$
⑦ $\triangle ADE \cong \triangle BCE$	⑦ HL $\cong$ HL
⑧ $\overline{AE} \cong \overline{BE}$	⑧ CPCTC
⑨ $\triangle AEB$ is an isosceles $\triangle$	⑨ An isosceles $\triangle$ has 2 $\cong$ legs

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?



$\tan 16.5 = \frac{X}{13.5}$

$X = 3.998882182$

$+4.5$

$\boxed{8.5}$

Find the volume of the inside of the pool to the *nearest cubic foot*.

$V_{\square\,prism} + V_{\triangle\,prism} + V_{\square\,prism}$

$lwh + (\frac{1}{2}bh)h + lwh$

$(22.5)(16)(4.5) + (\frac{1}{2}\cdot 13.5)(3.998882182)(16) + (12.5)(16)(8.498882182)$

$1620 + 431.8792757 + 1699.776436$

$\boxed{3752}$

Question 36 is continued on the next page.

Question 36 continued

A garden hose is used to fill th pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to th *nearest hour,* will it take to fill the pool 6 inches from the top? [1 ft^3 = 7.48 gallons]

1 ft = 12 in

.5 ft = 6 in

$V_{pool} = 3752$

$V_{air} = .5(16)(35) = 280$

$3752 - 280 = 3472\ ft^3(7.48\ g) = 25970.56\ g$

$$\frac{25970.56}{(10.5)(60)} = \frac{25970.56}{630} \approx \boxed{41\ hrs}$$